



TEACHER NOTES: THREE DIGIT TRICK

This trick could be used as a hook for an algebraic proof lesson.

Instructions for students:

Type a three-digit number into your calculator. Eg 723

Then type it again, so that you now have a six-digit number which is the same three digits repeated. Eg 723,723

Instructions for the 'mathemagician':

Point out to the students that there is a one in five chance (if they were choosing randomly), that their number is a multiple of 5.

Then ask them to divide their number by 7 to see if it is a multiple of 7. Ask them to put their hand up if their number is a multiple of 7. The students (given the set up above), are probably expecting about one seventh of the class to have made a multiple of 7. But everyone has made a multiple of 7!

Now ask them to divide their number by 11 to see if it is a multiple of 11. It is!

And ask them to divide their number by 13 to see if it is a multiple of 13. It is!

And ask them if there is anything else that they notice...they should be left with their starting number!

The key thing to note is that 7, 11 and 13 are the prime factors of 1001. And the number they started with is their three-digit number multiplied by 1001.

Algebraic proof

It can be proved algebraically that this trick will always work, as below.

$$\begin{aligned}abc, abc &= abc, 000 + abc \\abc, abc &= abc \times 1000 + abc \times 1 \\abc, abc &= abc \times (1000 + 1) \\abc, abc &= abc \times 1001 \\abc, abc &= abc \times 7 \times 11 \times 13\end{aligned}$$

This shows why you get back to abc if you divide abc, abc by 7, 11 and 13.