WRAPPING PRESENTS

Wrapping Presents Investigation involves finding lengths and area of shapes, problem solving and working systematically, but doesn't necessarily involve using algebra.

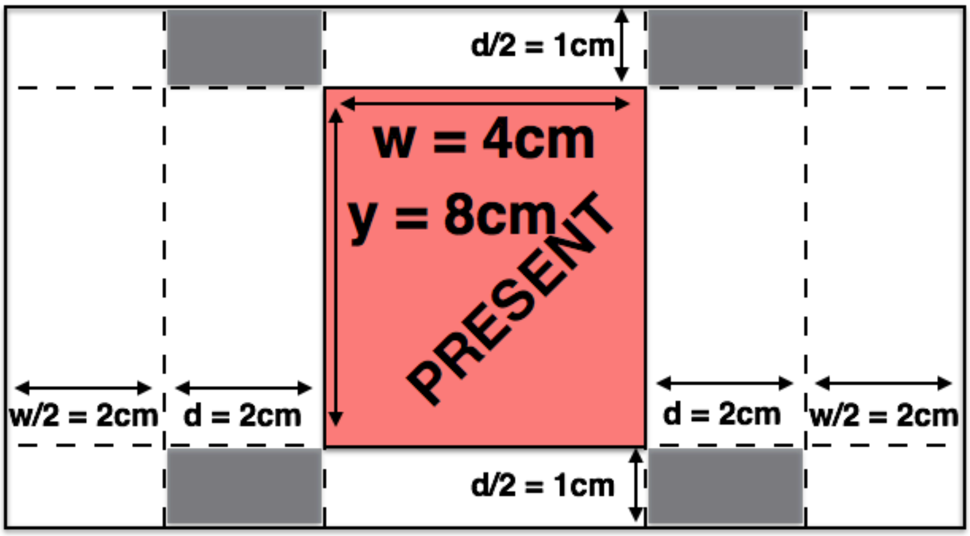
Wrapping Presents with Maths has three tasks. **Task 1 and 2** involve **GCSE** level material: use of algebra, Pythagoras' Theorem, surds. **Task 3** uses **A-Level** material: differentiation, solving polynomials.

See the solutions to the tasks below.

Wrapping Presents Investigation

Task 1

We've marked on missing lengths below



We can see the horizontal length is: **2 + 2 + 4 + 2 + 2 = 12cm**

We can see the vertical length is: **8 + 1 + 1 = 10cm**

Area of the rectangular piece of wrapping paper is: **120cm^2**

Task 2

Students can draw any cuboid with volume of **64cm³**

Task 3

The list of cuboids with whole number dimensions and a volume of **64cm³** are listed below, with the area of wrapping paper for each also listed. The cuboids that minimise the wrapping paper area are in bold.

Some students might find the general formula to help them more quickly find the wrapping paper area needed.

The general formula is:

Area = $2\,(d+y)(d+w)$

Depth (d) in cm	Width (w) in cm	Length (y) in cm	Wrapping paper area in cm^2
16	2	2	2 × 18 × 18 =648
2	16	2	2 × 4 × 18 =144
2	2	16	2 × 18 × 4 =144
8	4	2	2 × 10 × 12 =240
8	2	4	2 × 12 × 10 =240
4	8	2	2 × 6 × 12 =144
4	2	8	2 × 12 × 6 =144
2	**8**	**4**	**2 × 6 × 10 =120**
2	**4**	**8**	**2 × 10 × 6 =120**
4	4	4	2 × 8 × 8 = 128

Wrapping Presents with Maths

Task 1:

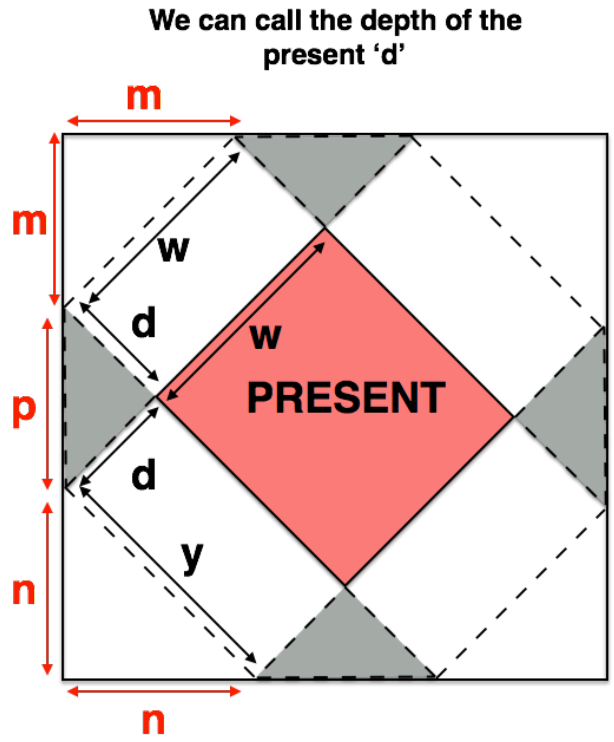
See the labelled diagram above.

The general formula is:

Area = $2\,(d+y)(d+w)$

Task 2:

See the diagram below with lengths labelled.



Using Pythagoras' Theorem:

$$2m^2 = w^2$$

$$m = \frac{w}{\sqrt{2}}$$

Similarly:

$$2n^2 = y^2$$

$$n = \frac{y}{\sqrt{2}}$$

And:

$$p^2 = 2d^2$$

$$p = \sqrt{2}d$$

Area of square = $\left(\frac{w}{\sqrt{2}} + \frac{y}{\sqrt{2}} + \sqrt{2}d\right)^2$

Task 3 (a)

$$\text{Area} = 2\,(d + y)(d + 6)$$

As the volume remains fixed (108cm^3) and we know the width, we can write the length in terms of the depth.

$$y = \frac{108}{6\times d}$$

Substituting this into the formula for the area, we can now express the area as a function of d.

$$\text{Area} = 2\,(d + \tfrac{18}{d})(d + 6)$$

$$= 2(d^2 + 18 + 6d + \frac{108}{d})$$

To find which value of d minimises the area, we differentiate the area with respect to d:

$$\text{Differential of the Area} = 2(2d + 6 - \tfrac{108}{d^2})$$

Find the value of d when the differential is zero:

$$0 = 2d + 6 - \tfrac{108}{d^2}$$

$$2d + 6 = \tfrac{108}{d^2}$$

$$2d^3 + 6d^2 - 108 = 0$$

Solving this equation finds the depth of the cuboid Zoe should make.

b)
Try substituting different factors of 108 into the equation above until we get zero as the output. We find this works for d=3.

$$2\times 3^3 + 6\times 3^2 - 108 = 0$$

The cuboid had width = 6cm, depth = 3cm and we need the volume to be 108cm^3, so the length is 6cm.

c)

Since the wrapping paper is square with method 2, we are looking to minimise its side length.

$$\text{Side length} = \frac{6}{\sqrt{2}} + \frac{y}{\sqrt{2}} + \sqrt{2}d$$

$$\text{Side length} = \sqrt{2}(3 + \frac{y}{2} + d)$$

As the volume remains fixed (108cm^3) and we know the width, we can write the length in terms of the depth.

$$y = \frac{108}{6\times d}$$

Substituting this into the formula for the side length, we can now express the side length as a function of d.

$$\text{Side length} = \sqrt{2}(3 + \frac{9}{d} + d)$$

To find which value of d minimises the side length, we differentiate the side length with respect to d:

$$\text{Differential} = \sqrt{2}(\frac{-9}{d^2} + 1)$$

Find the value of d when the differential is zero

$$\frac{9}{d^2} = 1$$

$$9 = d^2$$

As d must be positive:

$$d = 3$$

The cuboid had width = 6cm, depth = 3cm and we need the volume to be 108cm^3, so the length is 6cm.

This is the same cuboid as when wrapping with method 1.

Now we can compare the area of paper that is needed for both method to wrap that cuboid:

Area of the paper with method 2:

$$\text{Area} = (\sqrt{2}(3 + 3 + 3))^2$$
$$= 2 \times 9^2$$
$$= 162 \text{ cm}^2$$

Area of the paper with method 1:

$$\text{Area} = 2\ (3 + 6)(3 + 6)$$
$$= 2 \times 9 \times 9$$
$$= 162 \text{ cm}^2$$

It does not matter which wrapping method Zoe uses! Both are as efficient as each other for wrapping the marzipan.