



TUBE KNOTS TEACHER NOTES

This exercise is about making knots by following paths on the London Underground. It is based on a Stand-up Maths video where Matt and friends find a Trefoil on the London Underground.

Different types of knots

Part of the challenge is working out which knot your path makes. Every type of knot can be 'projected' (arranged) in multiple ways, and in order to identify which knot we have made we need to re-arrange it into its simplified version.

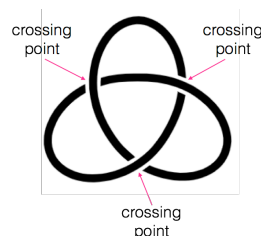
For example, the least complex knot (called the 'unknot') is just a loop, but it can be laid down/arranged so it looks like a figure of eight. This means if we trace a figure of eight path on the underground we have made an 'unknot' - a string version of our path could be re-arranged/simplified into a single loop.

We classify knots by their minimum number of crossing points – that is, the number of crossing points they have when they are in their simplest form. The minimum number of crossing points for the 'unknot' is zero as it is a loop in its simplest form, although as we have seen it can be re-arranged to have more crossing places (the figure of eight projection has one crossing place).

For more complex knots this process of re-arranging/simplifying can be quite tricky, and a set of moves called the Reidemeister moves can be employed to do this. Read more about the Reidemeister moves here:

<http://mathworld.wolfram.com/ReidemeisterMoves.html>

The unknot is the simplest knot, but it's trivial. The Trefoil knot is the simplest non-trivial knot. The minimum number of crossing places in the Trefoil is three, as shown below.



We also call it the 3_1 knot. The 3 here represents the minimum number of crossing points.



Trefoils on the Underground

We can trace out the path of a Trefoil by travelling on the London Underground!

If we start in one place and end up back in that same place, with our path crossing over itself three times (alternating between over and under – as in the diagram above), then we have travelled on a Trefoil-shaped path.

See the route below that Matt and friends travelled on in the video. Start at King's Cross St Pancras and follow the path with your pencil until you get back to King's Cross. Can you see how this is a Trefoil? A line with) (at its ends before a junction indicates that line is going **underneath** the other line.

You may need to make the knot in string to convince yourself that it can be simplified it so it has only three crossing places.



Note: This crossing point would be undone when we re-arrange/ simplify the knot.

Task for students

The **challenge for students** is to see if they can find their own Trefoil knots, or other more complex knots, on the London Underground.

We've included a **Map of the London Underground for Central London**. This is a cropped version of the full underground map – most knots can be found just using this central area. Alternatively, a full Underground map can be found here: <http://content.tfl.gov.uk/standard-tube-map.pdf>



Students could cover the map with tracing paper and trace paths on that. Alternatively, if students have a laminated tube map (or put their tube map in a plastic wallet) they could draw paths with a dry wipe pen that would rub off.

Once students have traced a path that they think might be a knot, they should make their path in string and simplify it if needs be.

Students will need to know which line is above/underneath at any crossing points (to know which bit of string is on top). Unfortunately, the tube map does not show which line is above/below at underground stations, so we've included the **Tube Crossing Points** document that gives them this information. **Note:** This crossing points information does not include information about the DLR and Overground lines. Students should stick to just using the other underground lines for the path of their knot.

Once students have made their knot in string and simplified it, they can draw the simplified ('smoothed out') version of their knot like Matt does in the video.

They can then look at our **Knots Information** document to work out what type of knot they have found. In the video, Henry says the most complex knot they found on the London Underground is the 5_2 . Can your students find a more complex one?

We advise showing students the video before they try to find their own knots on the London Underground.