TEACHER SOLUTIONS

International Space Station

Firstly, r is the distance from the centre of Earth to the space station, so we need to sum the radius of the Earth and the altitude of orbit.

$$r = 6{,}000 + 420 = 6420\ km$$

Note that the formula given requires r to be **in metres,** so we then convert this distance to metres.

$$r = 6{,}420{,}000\ m$$

Then using the formula:

$$v^2 = GM\left(\frac{1}{6{,}420{,}000}\right)$$

$$v^2 = 6.7 \times 10^{-11} \times 6.0 \times 10^{24} \times \left(\frac{1}{6{,}420{,}000}\right)$$

$$v^2 = 62{,}616{,}822.42990$$

$$v \approx 7{,}913\ ms^{-1}$$

$$v \approx 8000\ ms^{-1}$$

$$v \approx 8\ kms^{-1}$$

BONUS:

A more accurate value of the radius of the Earth is 6378km. Therefore:

$$r = 6798km$$

Let's use this more accurate figure to re-calculate v.

$$v^2 = 6.7 \times 10^{-11} \times 6.0 \times 10^{24} \times \left(\frac{1}{6{,}798{,}000}\right)$$

$$v^2 = 59135039.71756$$

$$v \approx 7689.93\ ms^{-1}$$

We could find the time it takes for the ISS to do one orbit by assuming the orbit is exactly circular and finding the circumference of that circle.

$$C = 2\pi r$$

And we know that:

$$time = \frac{distance\ travelled}{speed}$$

$$time\ of\ orbit = \frac{2\pi r}{v}$$

Substituting the more accurate values in:

$$time = \frac{13{,}596{,}000\pi}{7689.93}$$

$$time \approx 5554\ s$$

$$time \approx 92.6\ mins$$

Students might use the more accurate value of r only for the calculation of the circumference, and not for the calculation of the speed. This would give them a value around 90 minutes. A conversation could be had about where the inaccuracy is sneaking in.

How Long is a Mars Year?

The same method as above can be used to find the length of planet years. Students could find a value for the speed and then use that to find the time of orbit.

Alternatively, we could begin by deriving a formula for the time of orbit, as follows.

Using the derivation above:

$$time\ of\ orbit = \frac{2\pi r}{v}$$

Using the speed formula:

$$v^2 = GM\frac{1}{r}$$

$$time\ of\ orbit = \frac{2\pi r}{\sqrt{GM\left(\frac{1}{r}\right)}}$$

$$time\ of\ orbit = \frac{2\pi r^{\left(\frac{3}{2}\right)}}{\sqrt{GM}}$$

Note again that the value of r needs to be **in metres,** and that distances have been given in kilometres in the activity.

Substituting the values for r in to the formula gives the length of a year for each of the planets (and Pluto), as listed below.

Planet	Orbit length in Earth days
Mercury	87.77
Venus	224.18
Earth	364.42
Mars	685.29
Jupiter	4326.62
Saturn	10809.67
Uranus	30660.78
Neptune	60021.30
Pluto	90403.30

The results are slightly inaccurate (as is noticeable for Earth) because we have assumed the orbits are circular and have used the semi-major axis of the elliptical shapes of those orbits as the radii of the circles.

A Pluto year is:

$$\frac{90403.30}{365.25} \approx 247.51\ earth\ years$$

Pluto was only officially a planet for 76 years from 1930 to 2006. Therefore, it was only officially a planet for about a third of a Pluto year.