



PI SUM SOLUTIONS

A)

$$\begin{aligned}\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} \dots &= \frac{1}{(2 \times 1)^2} + \frac{1}{(2 \times 2)^2} + \frac{1}{(2 \times 3)^2} \dots \\ &= \frac{1}{2^2} \left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \dots \right] \\ &= \frac{1}{4} \left[\frac{\pi^2}{6} \right] \\ &= \frac{\pi^2}{24}\end{aligned}$$

B)

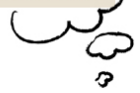
$$\begin{aligned}1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots &= \frac{\pi^2}{6} - \frac{1}{4} \left[\frac{\pi^2}{6} \right] \\ &= \frac{3}{4} \left[\frac{\pi^2}{6} \right] \\ &= \frac{\pi^2}{8}\end{aligned}$$

C)

$$\begin{aligned}\frac{1}{3^2} + \frac{1}{6^2} + \frac{1}{9^2} + \dots &= \frac{1}{3^2} \left[\frac{\pi^2}{6} \right] \\ &= \frac{1}{9} \left[\frac{\pi^2}{6} \right]\end{aligned}$$

and

$$\begin{aligned}\frac{1}{4^2} + \frac{1}{8^2} + \frac{1}{12^2} + \dots &= \frac{1}{4^2} \left[\frac{\pi^2}{6} \right] \\ &= \frac{1}{16} \left[\frac{\pi^2}{6} \right]\end{aligned}$$



and

$$\begin{aligned} \frac{1}{12^2} + \frac{1}{24^2} + \frac{1}{36^2} + \dots &= \frac{1}{12^2} \left[\frac{\pi^2}{6} \right] \\ &= \frac{1}{144} \left[\frac{\pi^2}{6} \right] \end{aligned}$$

$$\begin{aligned} 1 + \frac{1}{2^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{10^2} \dots &= \frac{\pi^2}{6} - \frac{1}{9} \left[\frac{\pi^2}{6} \right] - \frac{1}{16} \left[\frac{\pi^2}{6} \right] + \frac{1}{144} \left[\frac{\pi^2}{6} \right] \\ &= \frac{\pi^2}{6} \left(1 - \frac{1}{9} - \frac{1}{16} + \frac{1}{144} \right) \\ &= \frac{\pi^2}{6} \left(1 - \frac{24}{144} \right) \\ &= \frac{\pi^2}{6} \left(\frac{120}{144} \right) \\ &= \frac{20\pi^2}{144} \end{aligned}$$

D

Write a sum that is equal to $\frac{4\pi^2}{25}$

$$\frac{1}{5^2} + \frac{1}{10^2} + \frac{1}{15^2} + \dots = \frac{1}{5^2} \left[\frac{\pi^2}{6} \right]$$

$$\frac{1}{5^2} + \frac{1}{10^2} + \frac{1}{15^2} + \dots = \frac{1}{25} \left[\frac{\pi^2}{6} \right]$$

The original sum with every fifth term removed:

$$\begin{aligned} \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{6^2} \dots &= \left(1 - \frac{1}{25} \right) \left[\frac{\pi^2}{6} \right] \\ &= \left(\frac{24}{25} \right) \left[\frac{\pi^2}{6} \right] \\ &= \frac{4\pi^2}{25} \end{aligned}$$