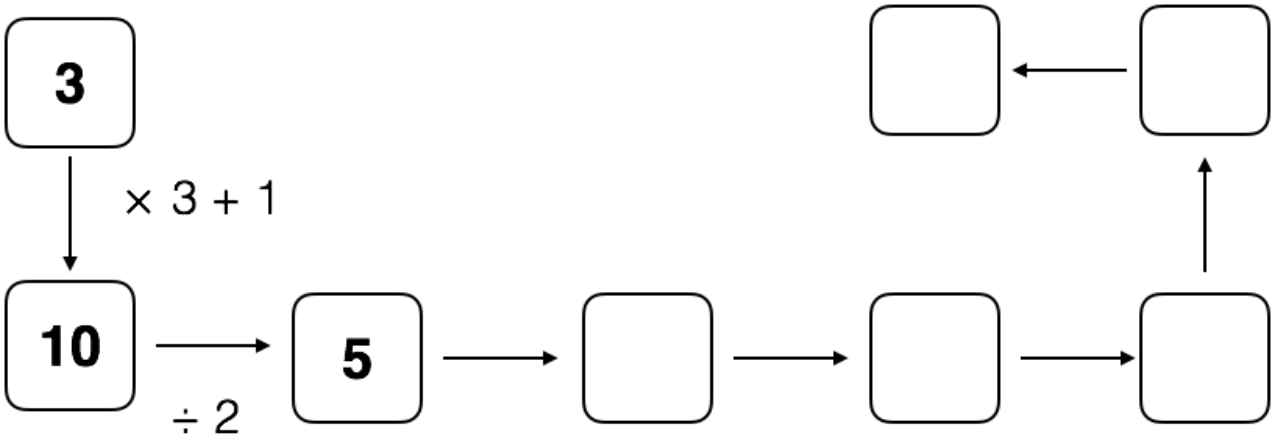
DRAW A COLLATZ MAP

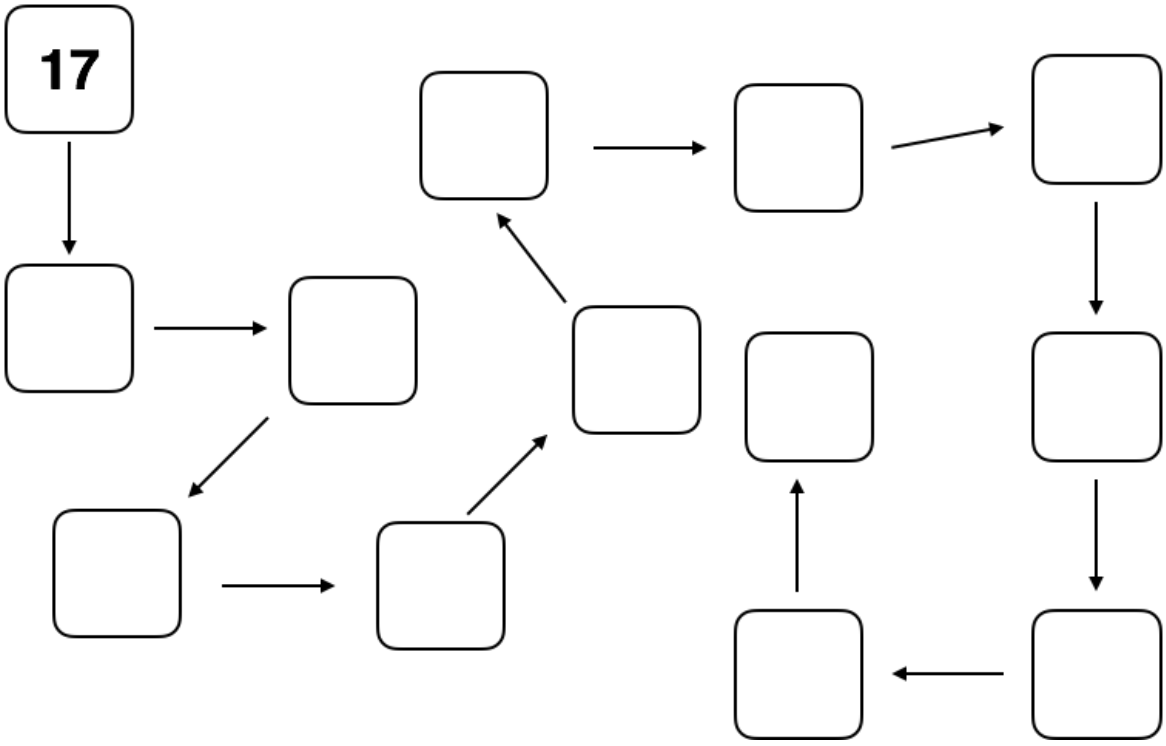
Complete the maps below by following these rules.

If the number in the box is:

- **EVEN, then divide it by two**
- **ODD, then multiply it by three and add one**

Write the number you get in the next box.
Continue until you get to 1!





TEACHER NOTES

Students do the activities on the worksheet **Draw a Collatz Map**. Possible further discussion questions and investigation ideas below.

Q: **What would happen if you continue the algorithm after you get to 1**?

You go back up to 4, then down to 2, then 1 again, and are stuck in an endless cycle.

Q: **Can you represent this on the map above without drawing any new boxes?**

This cycle could be represented by drawing an arrow going from 1 to 4.

Q: **What similarities are there between the two maps?**

Students notice that the final 7 numbers in both maps is the same. After 10 both maps follow the same path to 1.

Tell students about the Collatz Conjecture:

The Collatz Conjecture says that all positive integer starting numbers will eventually go down to 1 if following the Collatz algorithm.

This conjecture has not yet been proven! It's an 'open problem' in mathematics - a problem that lots of mathematicians around the world are currently trying to solve. Maybe one day you will be the one to do it and prove the conjecture!

Investigate how many steps it takes numbers to get to 1.

Students could draw maps for all the numbers below 10, and record how long it takes them to get to 1 in each case.

The spreadsheet entitled '**How Many Steps'** (can be downloaded from the site) shows how many steps each number under 1000 takes to get to 1.

Students may notice:

Powers of 2 are relatively quick because they only get divided by 2 and only ever get smaller.

9 takes the highest number of steps (for numbers under 10), taking 19 steps (20 boxes).

Q. **All maps we've tried end with a similar path. What is the longest sequence of numbers/path that all maps must end with? (Assuming the Collatz Conjecture is true and all maps do go to 1).**

Work backwards from 1.

The only way (under the Collatz algorithm) to get to 1 is from 2, because no positive integer multiplied by 3, plus 1, gives us 1.

The same logic shows that the only way to 2 is from 4, and so on, until we conclude that all maps must end with 16,8,4,2,1.

But 16 can be made by either halving 32, or from 5. So, there are two routes into 16.

*See Worksheet '**Draw a Collatz Super Map'** for more on this.*

THE GREAT COLLATZ COLLAB

Students could now take part in **The Think Maths Great Collatz Collab**

Students make their own map for a **three-digit number of their choice** and teachers scan and submit it to us. We are cropping small sections of maps we receive to make a giant collaborative 'super map' (a map that combines lots of maps together).

All info in the document 'The Great Collatz Collab' that can be downloaded from: **www.think-maths.co.uk/collatz-collab**

Deadline for submissions **Wednesday September 28 2022**