TEACHER SOLUTIONS

Rocket Energy

We need to divide the kinetic energy by the gravitational potential energy.

We can divide both the numerator and denominator by m.

$$\frac{\frac{1}{2}mv^2}{mgh} = \frac{\frac{1}{2}v^2}{gh}$$

$$= \frac{\frac{1}{2}\times(8000)^2}{9.81 \times 408{,}000}$$

$$= \frac{\frac{1}{2}\times(8000)^2}{9.81 \times 408{,}000}$$

$$\approx 8.00$$

The kinetic energy is 8 times greater than the gravitational potential energy.

The Rocket Equation - Bonus activity

$$\frac{m_f}{m_0} \approx 0.102$$

m_f = 140,000, m_0 = 140,000 + m_{fuel}

$$\frac{140{,}000}{140{,}000 + \ m_{fuel}} \approx 0.102$$

$$140{,}000 \approx 0.102(140{,}000 + m_{fuel})$$

$$140{,}000 \approx 0.102 \times 140{,}000 + 0.102 \times m_{fuel}$$

$$\frac{140{,}000 - (0.102 \times 140{,}000)}{0.102} = \ m_{fuel}$$

m_{fuel} ≈ 1,232,549kg

That’s the fuel needed for low Earth orbit.

For escape velocity:

$$\frac{m_f}{m_0} = e^{-\frac{11,190}{3500}}$$

$$\frac{m_f}{m_0} = 0.041$$

m_f = 140,000, m_0 = 140,000 + m_{fuel}

$$\frac{140,000}{140,000 + m_{fuel}} \approx 0.041$$

$$140,000 \approx 0.041(140,000 + m_{fuel})$$

$$140,000 \approx 0.041 \times 140,000 + 0.041 \times m_{fuel}$$

$$\frac{140,000 - (0.041 \times 140,000)}{0.041} = m_{fuel}$$

$m_{fuel} \approx$ 3,274,634

So, the additional fuel required to reach escape velocity is:

$$3,274,634 - 1,232,549 = 2,042,085\text{kg}$$