FROG JUMPS SOLUTIONS

How many ways?

If students follow the 'Hint' questions in the **Frog Jumps Task** document, they should find that the number of ways of crossing the pond doubles each time another lily pad is added. There are:

- 2 ways for 1 lily pad
- 4 ways for 2 lily pads
- 8 ways for 3 lily pads

By following the pattern (raising 2 to the power of the number of lily pads) students can find that there are $\mathbf{2^9 = 512}$ ways to cross the pond with nine lily pads.

This pattern can be explained by noticing that each lily pad can either be stepped on, or not. So, with two options per lily pad, the number of different options for n lily pads is $\mathbf{2^n}$.

How many steps?

Students could be encouraged to explore if there is also a pattern in the number of steps the different journeys take.

Students may have already recorded how many of the journeys took two steps, or three steps, etc. when recording the journeys.

Alternatively, we've made a results table (**Frog Results** document) that could be used by students to record their results. We've included the solutions for this table as the final page of this document.

Students' results should end up making Pascal's triangle! With each row summing to successive powers of two, as in Pascal's Triangle.

Students could be asked:

- Can you see any patterns in the table of results?
- Can you explain why these patterns occur?

The appearance of Pascal's Triangle

Summing pattern:

In the results table, any two cells that are adjacent in the same row (like the two circled cells below) add to the cell below the right hand of the pair of cells (the cell with the triangle below).

	one step	two steps	three steps	four steps	five steps
0 Lily Pads	1				
1 Lily Pad	1	1			
2 Lily Pads	1	2	1		
3 Lily Pads	1	3	3	1	
4 Lily Pads	1	4	6	4	1

These cells appear as below when Pascal's Triangle is presented in its usual way.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

This pattern can be explained in the Frog Problem as follows:

Let's imagine we want to find how many ways there are of crossing a pond with **four lily pads**, in **three jumps** (the entry with the triangle in the table above).

All lily pads can either be landed on by the frog, or not. So, when the **fourth lily pad** is added it can either be landed on, or not.

The number of ways of getting to the other side in three jumps if the **fourth lily pad is NOT used**, is equal to the number of ways of crossing the **three lily pad** pond in **three jumps** (we've just made the final jump a bit longer). This is the right hand circled cell above.

And the number of ways of getting to the other side if **the fourth lily pad is used** is equal to the number of ways of crossing the **three lily pad** pond in **two jumps** (the second jump becomes the second and third jump when it is broken by the frog landing on the fourth lily pad). This is the left hand circled cell above.

Therefore, summing the two circled cells gives us the value of the cell below them (with the triangle). This also applies to all other cells grouped in the same way.

Choose function values:

Let's imagine we want to find how many ways there are of crossing a pond with **four lily pads**, in **three jumps**.

Making three jumps means landing on two different lily pads. There are ${}^{4}C_{2}$ ways of choosing which two of the four lily pads to land on, so there are ${}^{4}C_{2}$ ways of crossing the **four lily pad** pond in **three jumps**. Generally, there are ${}^{p}C_{j-1}$ ways of crossing a pond with **p** lily pads, in **j** jumps.

This matches up the values Pascal's Triangle gives us (shown below).

	one step	**two steps**	**three steps**	**four steps**	**five steps**
0 Lily Pads	${}^{0}C_{0}$				
1 Lily Pad	${}^{1}C_{0}$	${}^{1}C_{1}$			
2 Lily Pads	${}^{2}C_{0}$	${}^{2}C_{1}$	${}^{2}C_{2}$		
3 Lily Pads	${}^{3}C_{0}$	${}^{3}C_{1}$	${}^{3}C_{2}$	${}^{3}C_{3}$	
4 Lily Pads	${}^{4}C_{0}$	${}^{4}C_{1}$	${}^{4}C_{2}$	${}^{4}C_{3}$	${}^{4}C_{4}$

FROG JUMP RESULTS

	1 step	2 steps	3 steps	4 steps	5 steps	6 steps	7 steps	8 steps	9 steps	10 steps	Total ways
0 lily pads	1										1
1 lily pads	1	1									2
2 lily pads	1	2	1								4
3 lily pads	1	3	3	1							8
4 lily pads	1	4	6	4	1						16
5 lily pads	1	5	10	10	5	1					32
6 lily pads	1	6	15	20	15	6	1				64
7 lily pads	1	7	21	35	35	21	7	1			128
8 lily pads	1	8	28	56	70	56	28	8	1		256
9 lily pads	1	9	36	84	126	126	84	36	9	1	512