



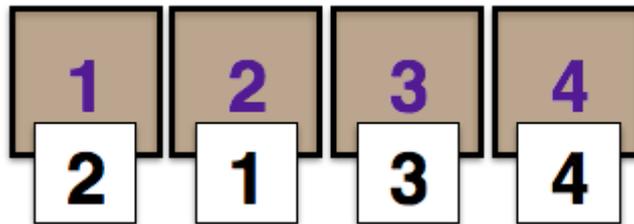
# THE PRISONER PROBLEM

## The problem

Four prisoners are numbered 1 to 4.

A prison warden puts four cabinets, also numbered 1 to 4, in a locked room.

The warden then has four cards numbered 1 to 4. They shuffle the cards and place one in each of the four cabinets randomly. The example below shows one possible arrangement of the cards in the brown cabinets.



The warden tells the prisoners they may each enter the cabinet room one at a time and open at most two cabinets each.

If they all manage to open the cabinet with their own card in it, they all survive. If one or more of them fails to open the cabinet with their card in it, they all die.

They are not allowed to communicate (either verbally or by doing something to the cabinets/cards/room) once the game has started. They are however allowed to come up with a strategy beforehand.

## The strategy

The prisoners use the best possible strategy with is for each prisoner to start by opening the box with their number on it.

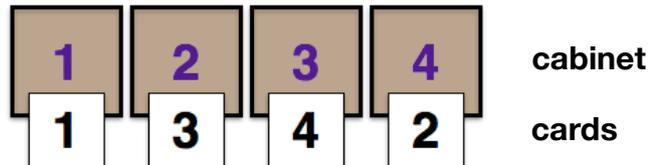
They then look inside and if it contains their card, they stop there – they have found their number.

However, if it contains a different number, they then open the box with that number on it.

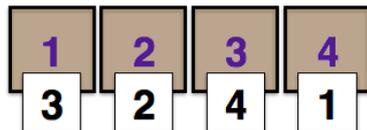
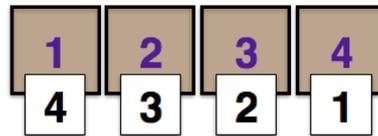
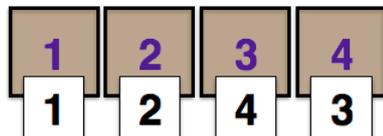


# THE PRISONER PROBLEM INVESTIGATION

Below is an example of one possible arrangement of the four cards in the cabinets.



- For this arrangement, work out which (if any) of the prisoners find their card using the strategy. Do the prisoners survive or die?
- For each of the arrangements below work out if the prisoners survive or die.



What do you notice about the arrangements where the prisoners survive? What about the arrangements where the prisoners die?

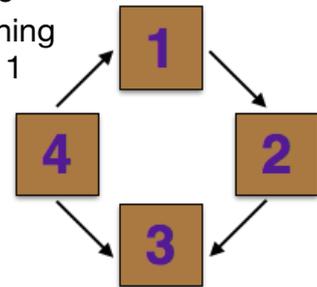
- Can you work out the probability that the prisoners survive using their strategy?



# THE PRISONER PROBLEM CYCLES

## 4-cycles

One way we can arrange the four cards in the cabinets is so that they form a **'4-cycle'**. In the example on the right, opening cabinet 1 sends the prisoner to cabinet 2 (because cabinet 1 contains card 2), and so on.



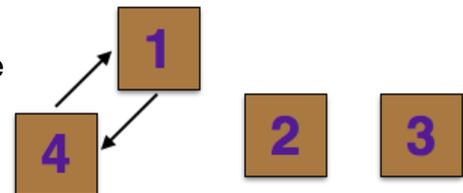
Whichever cabinet you start on in the cycle you will not find that card until you open the fourth and final cabinet. Since the prisoners are only able to open at most two cabinets, they will not find their cards and will die.

All arrangements that form 4-cycles are bad for the prisoners. **How many different possible 4-cycles can be made?**

## 2-1-1 formation

Another way we can arrange the four cards in the cabinets is so that they form a '2-1-1' formation.

In the example on the right 1 and 4 are in a 2-cycle together: opening cabinet 1 sends the prisoner to cabinet 4, and visa-versa. And cabinet 3 contains card 3, and cabinet 2 contains card 2.



In 2-1-1 formations like this the prisoners will survive because all of them will find their cards by opening one or two cabinets. **How many different possible 2-1-1 formations can be made?**

## Task:

By considering what other formations the cards could be in, can you find the probability that the prisoners survive using their strategy?