THE PRISONER PROBLEM SOLUTIONS

Solutions to 'The Prisoner Problem Investigation'

1) In this arrangement:
   -Prisoner 1 gets their card by opening their cabinet
   -Prisoner 2 opens cabinet 2 and gets card 3, then opens cabinet 3 and gets card 4, and then stops there. They do not find their card.
   -Prisoner 3 and 4 also do not find their card.
   Since at least one of them did not get their card, they all die.

2) Survive, Survive, Die, Die. See next page for what students might notice.

3) For this, students could write out all the different arrangements and test each one to see if the prisoners live or die.

   There are 24 arrangements ($4! = 4 \times 3 \times 2 \times 1$).

   These are listed below. Next to each one it is indicated if the prisoners survive or die. We see that 10 of the 24 arrangements lead to the prisoners surviving. There is a 42% chance the prisoners will survive with their strategy.

1	2	3	4	
1	2	3	4	✓
1	2	4	3	✓
1	3	2	4	✓
1	3	4	2	✗
1	4	2	3	✗
1	4	3	2	✓
2	1	3	4	✓
2	1	4	3	✓
2	3	1	4	✗
2	3	4	1	✗
2	4	1	3	✗
2	4	3	1	✗

The 'cycles' method

There is an alternative method.

In part 2, students might notice that arrangements that work are when cards are in their own cabinets or two cabinets have each other's cards in a straight swap.

If cards are in longer swap cycles than this – cycles containing 3 or 4 cards - any prisoners whose number is in the cycle will not get their card. This is because they would need to complete the cycle, opening 3 or 4 cabinets, before getting their card.

Students could use this insight to work out how many arrangements survive/die without writing them out. They could calculate how many arrangements would/would not involve 3 or 4 way swaps.

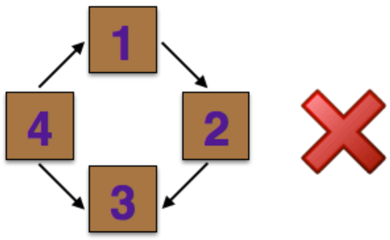
See our worksheet **'Prisoner Problem Cycles'** for more information about different possible cycles, and ideas for questions you could ask students. See how to calculate how many of each 'cycle type' there are in the **Prisoner Problem Solutions** on the page below.

Solutions to the 'The Prisoner Problem Cycles'

Our worksheet **'Prisoner Problem Cycles'** is for teachers who want to give students the cycle representation to start with.

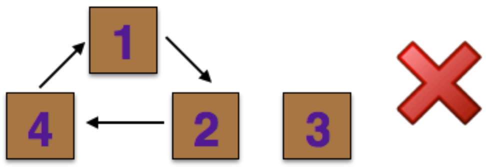
All different types of formation (including the 4-cycle and the 2-1-1 formation) are represented below next to a tick or cross to indicate if that formation would lead to the prisoners surviving or dying.

See also the calculation for how many different arrangements there are of each type.



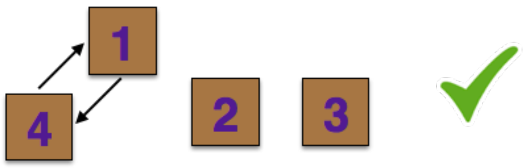
$3! = 6$

We can keep the 1 fixed. There are then 3! ways of choosing the positions of the other three numbers in the cycle.



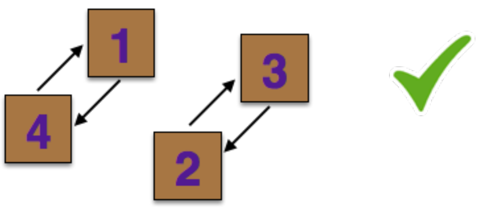
${}^4C_3 \times 2! = 8$

There are ${}^4C_3$ ways of choosing the three for the cycle. And then 2! ways of arranging that cycle.



${}^4C_2 = 6$

There are ${}^4C_2$ ways of choosing the two for the 2-cycle. The other two are then fixed.



${}^4C_2 / 2 = 3$

There are ${}^4C_2$ ways of choosing two numbers from four. However, since what is left over is also a pair, we need to half ${}^4C_2$ to avoid repeats.

${}^4C_4 = 1$

We see that 10 of the 24 arrangements lead to the prisoners surviving. There is a 42% chance the prisoners will survive with their strategy.